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Algorithms for Asymmetric GEMM-like Operations

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ADEPT

ADEPT End-of-project Presentation

Motivation

- Dense matrix-matrix multiplication (GEMM) is an intensely studied kernel. We have:
  - Shared Memory GEMM algorithms minimizing memory-to-processor communication through cache-aware tiling
  - Distributed Memory GEMM algorithms minimizing processor-to-processor communication (Cannon, SUMMA)
  - Accelerators optimized for GEMM computations (GPU, Google TPU, GEMMINI systolic array...)
- For the past three semesters, we've looked at GEMM-like operations where the input arguments have **unequal access costs**, both in distributed and shared memory. We summarize the work here.

Communication Avoiding SDDMM, FusedMM Operators

In review at IPDPS 2022

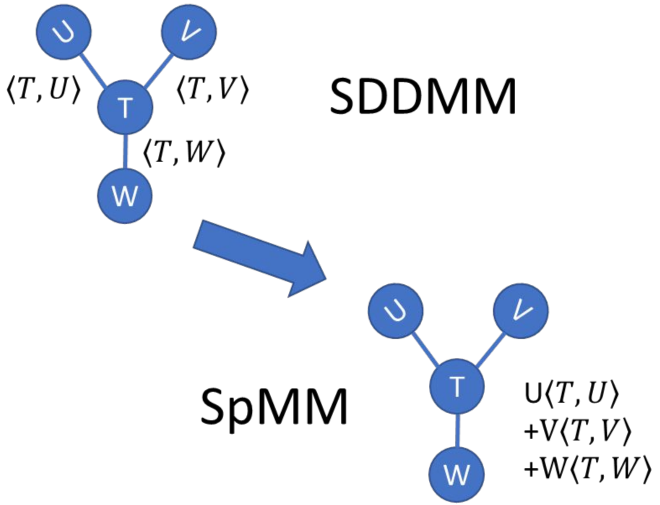
- Sparse-Dense matrix multiplication (SpMM) and Sampled Dense-Dense Matrix multiplication (SDDMM) are key computational kernels in graph learning, scientific computing operations

- For graphs, SDDMM is key kernel for **message generation**, SpMM is key for **message aggregation**

- Extensive existing work on shared memory optimization for both kernels and distributed-memory SpMM operations

$$\mathrm{SpMMA(S,\ B)} := S \cdot B$$

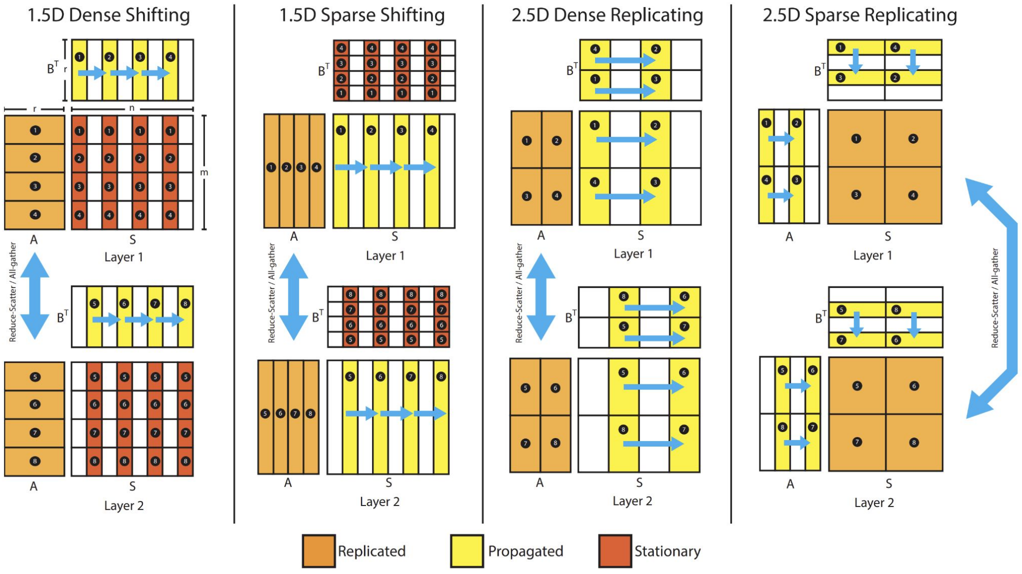
$$\mathrm{SDDMM}(A, B, S) := S * (A \cdot B^T)$$



Communication Avoiding SDDMM, FusedMM Operators

In review at IPDPS 2022

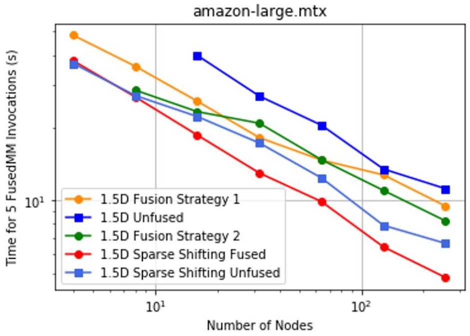
- We provide the **first** distributed-memory, communication-avoiding implementation for SDDMM using the same data distributions as SpMM Cannon-style algorithms
- Two distinct methods for combining SDDMM / SpMM primitives:
    - Reuse replication of input dense matrix
    - Overlap SDDMM / SpMM phases
- When additional memory is available and input dense matrix does not change, **even further communication savings are possible**

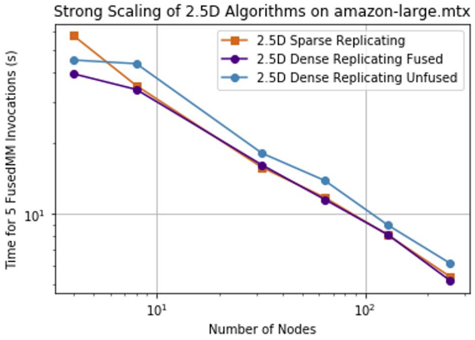


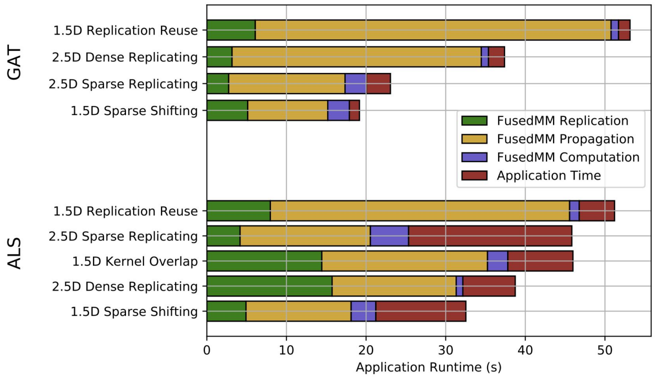
Communication Avoiding SDDMM, FusedMM Operators

In review at IPDPS 2022

- Experiments ran on 256 Knights Landing Cores on Cori, a Cray XC40 at LBNL
- **Combining the SDDMM and SpMM primitives (FusedMM) yielded significant communication savings. Up to 1.6x faster runtime for the pair of kernels on 256 nodes with kernel overlap.**
- Embedded our algorithms in applications. Tested **collaborative filtering problem (Netflix Challenge)** and **Graph Attention Network Training** on symmetric sparse matrices with tens of millions of rows, ~200-300 million edges







Dense-Times-Random, Mixed Precision MatMul

Work in Progress

- Now consider computations of the form $C = A \cdot B$, where A, B are m x k, k x n dense matrices and A is either:
  - Random i.i.d. (e.g. sketching operations in randomized linear algebra)
  - Lower precision than B or C (mixed-precision neural network training
- If A is i.i.d. random, suppose that it costs less to regenerate entries of A in registers than to load it from memory (assume a single level of caching for now)
- Could generate the random matrix and dispatch a GEMM cal for either problem. Can we take advantage of the unequal access cost of the two inputs?
  - In theory: yes!
  - In practice: working on it...

Weakened Memory Lower Bounds

Work in Progress

- Let *M* be the number of data words in the cache that either *B* or *C* can hold. Standard GEMM has a data movement lower bound:

$$\Omega\left(\frac{mnk}{\sqrt{M}}\right) + \Omega(\text{lower order terms...})$$

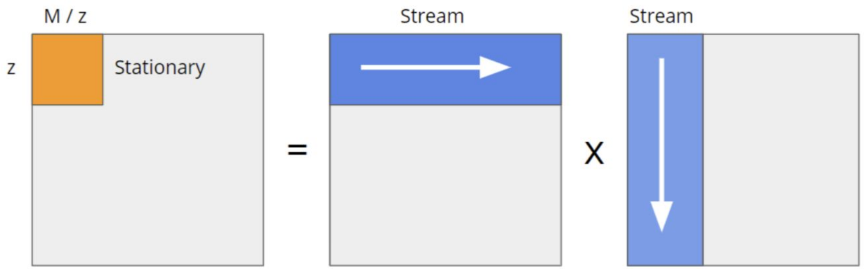
- If loading words of *A* costs $0 < \varrho < 1$ time compared to loading words of *B* or *C*, the lower bound **weakens** to:

$$\Omega\left(\frac{mnk\sqrt{\rho}}{\sqrt{M}}\right) + \Omega(\text{lower order terms...})$$

Tiling to Meet Lower Bounds

Work in Progress

- Specially tuned cache tiling shape for GEMM allows us to meet the lower bound in theory
- **In practice**: Hardware-accelerated RNG required to get performance fast enough to take advantage of tiling, need hardware support for mixed-precision matrix multiplication
- We are continuing to develop theory and experiments for unequal access cost GEMM



Thanks!